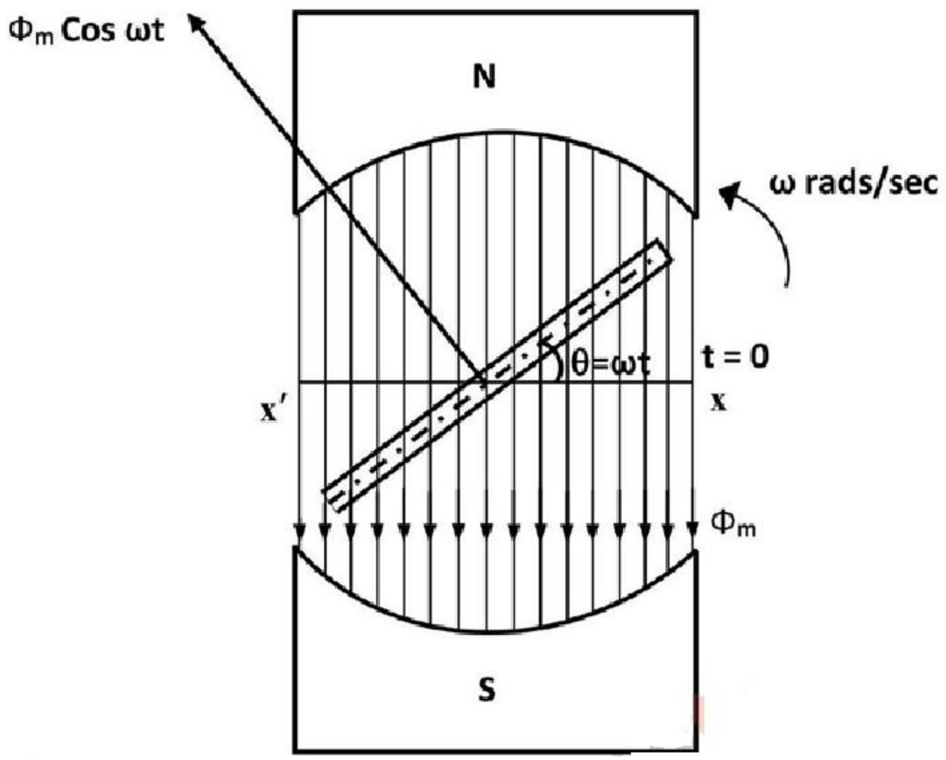
UNIT- 2
SINGLE PHASE AC CIRCUITS

Generation of sinusoidal AC voltage, definition of average value, R.M.S. value, form factor and peak factor of AC quantity, Concept of Phasor, Concept of Power factor, Concept of impedance and admittance, Active, reactive and apparent power, analysis of R-L, R-C, R-L-C series and parallel circuit.

Single Phase AC Circuits

Generation of single phase voltages:



Let us consider a coil with N turns, rotating in a uniform magnetic field with an angular velocity $\omega \text{rads/sec}$ as shown in figure. Let the time be measured from the x-axis. When the plane of the coil coincides with the x-axis maximum flux Φ_m links the coil. After time t the coil moves through an angle $\theta = \omega t$

In this position the component of the flux which is perpendicular to the plane of the coil is $\Phi = \Phi_m \text{Cos}\omega t$

We have flux linkages $= \mathrm{N\Phi} = \mathrm{N\Phi_m Cos\omega t}$; Induced emf at this instant is, $e = -\frac{d}{dt}(N\Phi)$

$$= -N\frac{d}{dt}(\Phi_m \mathrm{Cos\omega t}) = -\omega N\Phi_m(-\mathrm{Sin\omega t}) = \omega N\Phi_m \mathrm{Sin\omega t}$$

When the coil makes an angle $\theta = 90°, \mathrm{Sin}\theta = 1$, Hence, the emf induced in the coil is maximum, ie. $\mathrm{E_m}$

$\therefore E_m = \omega N\Phi_m$ or $e = E_m \mathrm{Sin}\omega t$ or $e = E_m \mathrm{Sin}\theta$

The induced emf varies as sine function of the time angle ωt and when emf is plotted against time a sine curve is traced.

Terms and Definitions:

Cycle: One complete set of positive and negative values of an alternating quantity is known as a cycle.

Frequency: The number of cycles/sec is called the frequency of the alternating quantity.

Time period: The time taken by an alternating quantity to complete one cycle is called its time period.

Amplitude: The maximum value either positive or negative of an alternating quantity is known as its amplitude.

Phase: It is the fraction of the time period of that alternating quantity which has elapsed since the current last passed through the zero position of reference.

Phase difference: It is the difference in phase angle between any two alternating quantities.

Instantaneous value: It is the value of any alternating quantity at a particular instant of time.

Average value: It is that value of steady current, which transfers across any circuit the same charge as is transferred by that alternating current during the same time. Average value, $\mathrm{I_{av}} = 0.637\mathrm{I_m}$

Root mean square value: It is given by that steady current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time. Root mean square value, $\mathrm{I_{rms}} = 0.707\mathrm{I_m}$

Form factor: It is defined as the ratio of its rms value and average value
Form factor $= rms$ value / average value $= 0.707\mathrm{I_m}/0.637\mathrm{I_m} = 1.11$ for sine wave .
Peak factor or Crest factor: It is defined as the ratio of its maximum value and rms value.
Peak factor $=$ Maximum value $/\mathrm{rms}$ value $= \mathrm{I_m}/0.707\mathrm{I_m} = 1.414$ for sine wave .
Resistance: Its property is to oppose the flow of current through it. Resistance is a measurable quantity and its unit is Ohm.

Inductance: Its property is to induce emf in itself whenever a changing current flows through it, its unit is Henry.

Inductive reactance: It causes opposition to the flow of current through it. Reactance is a non-measurable quantity which can only be calculated its unit is Ohm.

Capacitance: It is the capacity of any capacitor to store charge and its unit is Farad.
Capacitive reactance: It causes opposition to the flow of current through it. Reactance is a non-measurable quantity which can only be calculated its unit is Ohm.

Impedance: It is the total opposition due to the resistance as well as the reactance of the circuit to the flow of current. It is a non-measurable quantity which can only be calculated its unit is Ohm.

Admittance: Admittance is a measure of how much current is admitted in a circuit. Admittance (Y) is the inverse of impedance (Z). Admittance has its most obvious utility in dealing with parallel AC circuits. The unit of admittance is Siemens.

Active power: It is also called Average power or True power or Real power. It is the actual power which is dissipated in the resistance of the circuit $P = VI\mathrm{Cos}\Phi$ or $P = I^2R$, the unit is watts or KW.

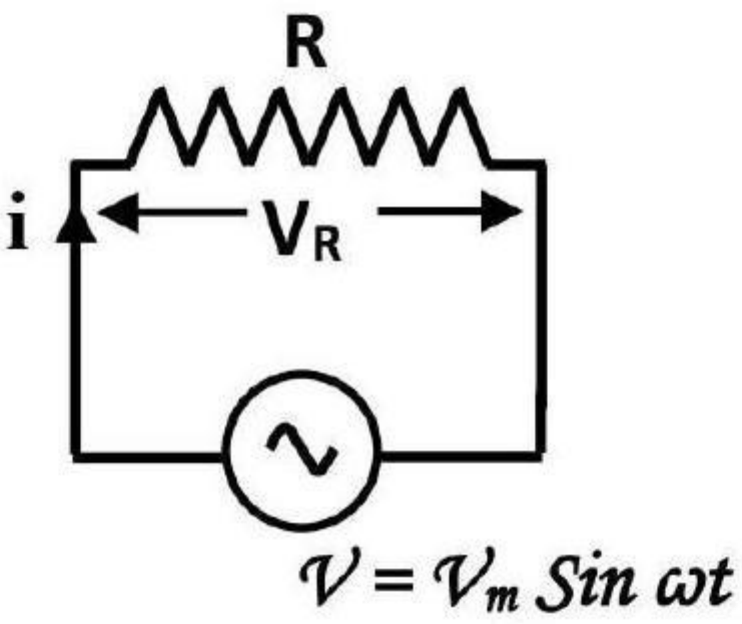
Reactive power: It is the power developed in the inductive reactance of the circuit. It is also called wattless power Reactive power $=$ VI $\mathrm{Sin}\Phi$, its unit is Reactive Voltamperes or Kilovoltamperes reactive

Apparent power: It is the product of the rms values of voltage and current.
Apparent power $= \mathrm{VI}$, its unit is Voltampere or Kilovoltamperes
Power factor: It is the cosine of the phase angle Φ existing between the voltage and current in any AC circuit. It has no unit. It can have values varying between zero and unity. It can also be either lagging or leading in nature.

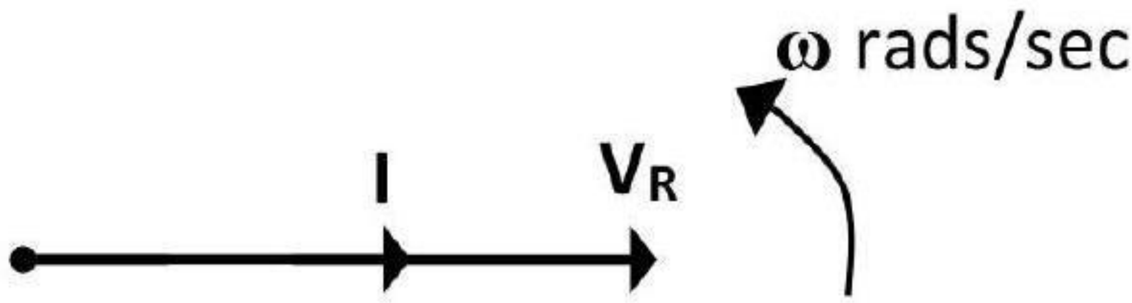
Phasor: A phasor is a complex number representing a sinusoidal function whose amplitude, angular velocity and initial phase are time invariant.

AC circuit with Resistance only:

Let the alternating voltage be $v = V_m \mathrm{Sin}\omega t$
the current $i = \frac{V_m}{R}$ or $i = \frac{V_m \mathrm{Sin}\omega t}{R}$
The value of i will be maximum when $\mathrm{Sin}\omega t = 1$
$\therefore I_m = \frac{V_m}{R}$ or $i = I_m \mathrm{Sin}\omega t$
The voltage across the resistor and the current through the resistor are in phase with each other.
The instantaneous power, $p = vi = V_m \mathrm{Sin}\omega t I_m \mathrm{Sin}\omega t$



CIRCUIT DIAGRAM



PHASOR DIAGRAM

$= V_m I_m \text{Sin}^2 \omega t = V_m I_m \left[\frac{1 - \text{Cos}2\omega t}{2}\right] = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\text{Cos}2\omega t$

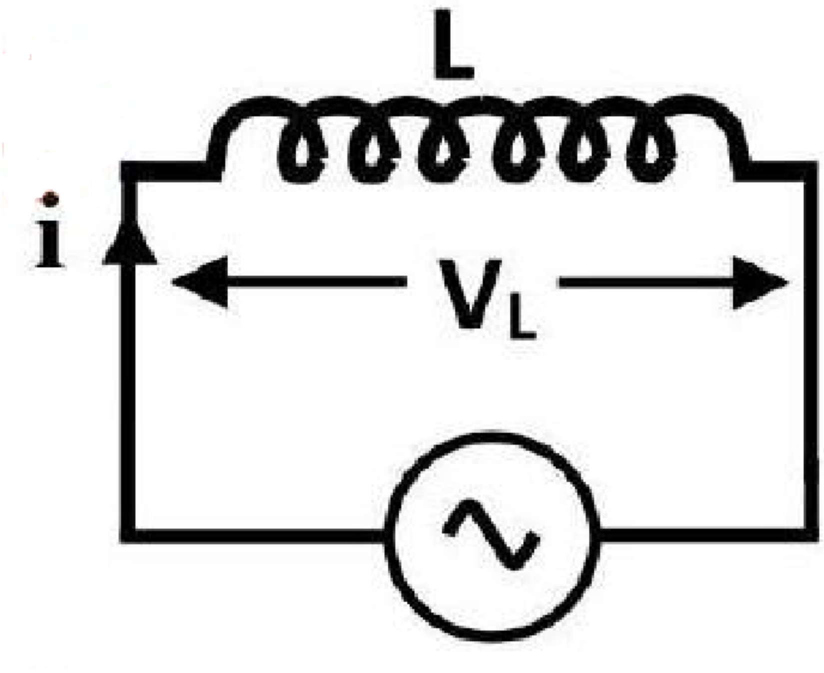
The average value of $\frac{V_m I_m}{2}\text{Cos}2\omega t$ over a complete cycle is zero

$\therefore P = \frac{V_m I_m}{2}$ or $= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$ $\therefore$ Power $= VI$ Hence, a pure resistive circuit consumes power.

AC circuit with Inductance only:

Let the alternating voltage be $v = V_m \text{Sin}\omega t$

the self induced emf, $e_L = L\frac{di}{dt}$



$\mathcal{V} = \mathcal{V}_m \text{Sin}\omega t$

CIRCUIT DIAGRAM

$\therefore V_m \text{Sin}\omega t = L\frac{di}{dt}$ or $di = \frac{V_m}{L}\text{Sin}\omega t dt$

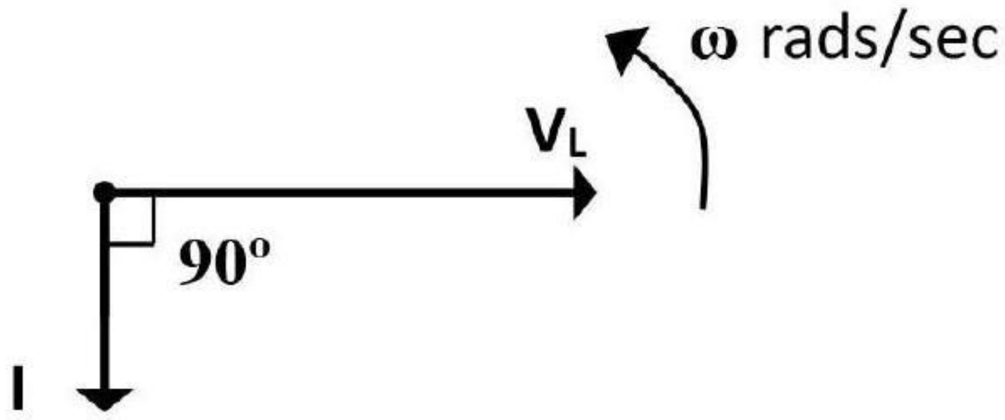
Integrating both sides, we have

$i = \frac{V_m}{L}\int \text{Sin}\omega t dt = \frac{V_m}{\omega L}[-\text{Cos}\omega t]$ or $i = \frac{V_m}{\omega L}\text{Sin}\left[\omega t - \frac{\pi}{2}\right]$

We have i to be maximum when $\text{Sin}\left[\omega t - \frac{\pi}{2}\right]$ is unity

$\therefore I_m = \frac{V_m}{\omega L}$ or $i = I_m \text{Sin}\left[\omega t - \frac{\pi}{2}\right]$

It is observed that the current lags the voltage by $90°$



PHASOR DIAGRAM

The quantity ωL is called inductive reactance and also represented as $X_L = 2\pi f L$

The instantaneous power, $p = vi = V_m \text{Sin}\omega t I_m \text{Sin}\left[\omega t - \frac{\pi}{2}\right] == -V_m \text{Sin}\omega t I_m \text{Cos}\omega t$

$= -V_m I_m \text{Sin}\omega t \text{Cos}\omega t \ = -\frac{V_m I_m}{2} \text{Sin} 2\omega t$

Power for the complete cycle, $P = -\frac{V_m I_m}{2} \int_0^{2\pi} \text{Sin} 2\omega t dt = 0$

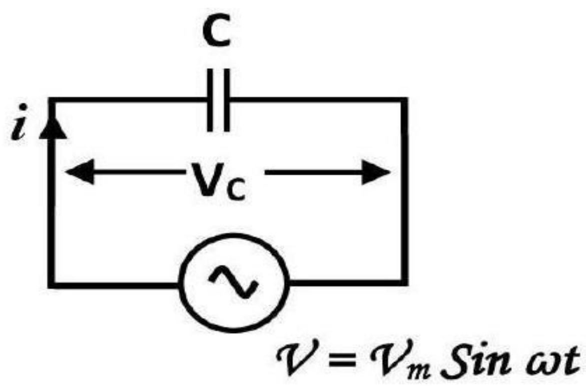
A pure inductive circuit does not consume any power

AC circuit with Capacitance only:

Let the alternating voltage be $v = V_m \text{Sin}\omega t$

Let charge on the plates be $= \boldsymbol{q}$

But charge, $\boldsymbol{q = Cv = CV_m \text{Sin}\omega t}$



Current, $i = \frac{dq}{dt} = \frac{d}{dt}[CV_m \text{Sin}\omega t] = \omega CV_m \text{Cos}\omega t$

or $i = \frac{V_m}{1/\omega C}\text{Cos}\omega t$ or $i = \frac{V_m}{1/\omega C}\text{Sin}\left[\omega t + \frac{\pi}{2}\right]$

The current i will be maximum when $\text{Sin}\left[\omega t + \frac{\pi}{2}\right]$ is unity

$\therefore I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_C}$ Where X_C is the capacitive reactance

$\therefore i = I_m \text{Sin}\left[\omega t + \frac{\pi}{2}\right]$ It is observed that the current leads the voltage by $90°$

The instantaneous power, $p = vi = V_m \text{Sin}\omega t I_m \text{Sin}\left[\omega t + \frac{\pi}{2}\right] = V_m \text{Sin}\omega t I_m \text{Cos}\omega t$

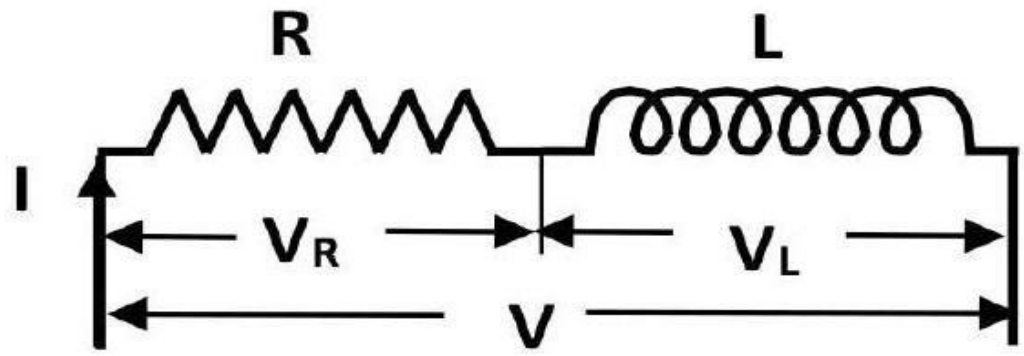
$= V_m I_m \text{Sin}\omega t \text{Cos}\omega t = \frac{V_m I_m}{2}\text{Sin}2\omega t$

Power for the complete cycle, $P = \frac{V_m I_m}{2}\int_0^{2\pi} \text{Sin}2\omega t dt = 0$

A pure capacitive circuit does not consume any power

R - L series circuit:

Let us consider a resistor and inductor in series.
If V is the rms value of the applied voltage then I



CIRCUIT DIAGRAM

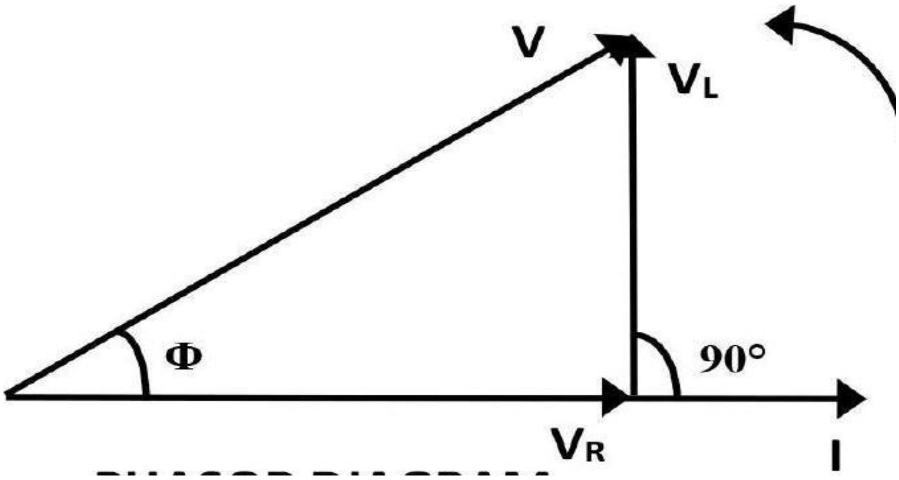
will be the rms value of the current drawn by the circuit.
The voltage across $R = V_R = IR$, where V_R is in phase with I.
The voltage across $\mathrm{L} = \mathrm{V_L} = \mathrm{IX_L}$, where $\mathrm{V_L}$ leads I by $90°$
The applied voltage **V** is the phasor sum of the two voltage drops $\mathrm{V_R}$ and $\mathrm{V_L}$
From the phasor diagram, we have

$V = \sqrt{{V_R}^2 + {V_L}^2} = \sqrt{IR^2 + {IX_L}^2} = I\sqrt{R^2 + {X_L}^2}$

$I = \frac{V}{\sqrt{R^2 + {X_L}^2}}$ or $I = \frac{V}{Z}$

Where Z is the impedance of the circuit $= \sqrt{R^2 + {X_L}^2}$

From the phasor diagram, $\tan\Phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$



PHASOR DIAGRAM

IMPEDANCE TRIANGLE

or $\Phi = \tan^{-1}\frac{X_L}{R}$ Also $\mathrm{Cos}\Phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \therefore \mathrm{Cos}\Phi = \frac{R}{Z}$

From the phasor diagram, we can draw the Impedance triangle as shown in figure.

We have Power, $P = VICos\Phi = [IZ]I\left[\frac{R}{z}\right] = I^2R$

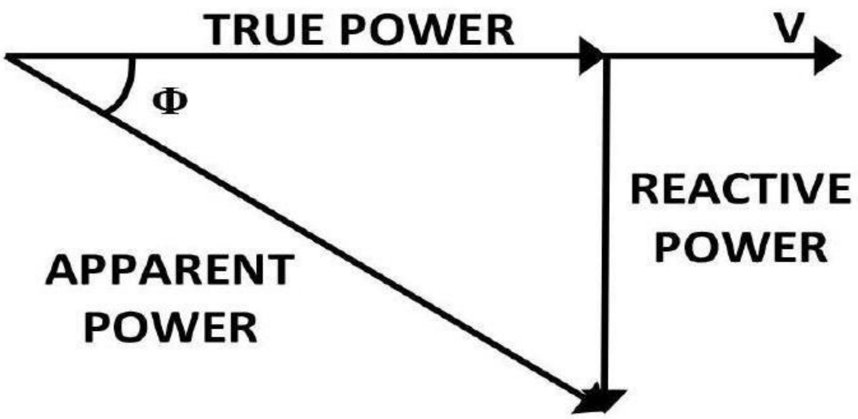
$\therefore$ Power $= I^2R$

From the power triangle, we have

True power $= P =$ VICos Φ

Reactive power $= P =$ VISin Φ

Apparent power $= P = VI$



R - C series circuit:

Let us consider a resistor and capacitor in series. If $\mathbf{V}$ is the rms value of the applied voltage then I will be the rms value of the current drawn by the circuit.

The voltage across $R = V_R = IR$, where V_R is in phase with I
The voltage across $\mathrm{C} = \mathrm{V_c} = \mathrm{IX}$, where $\mathrm{V_c}$ lags I by $90°$
The applied voltage $\mathbf{V}$ is the phasor sum of the two voltage drops V_R and V_C. From the phasor diagram, we have

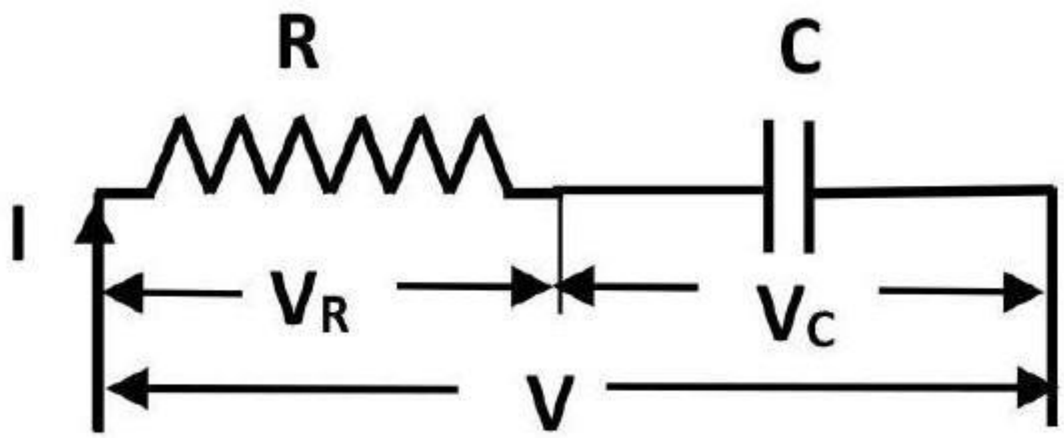
$V = \sqrt{{V_R}^2 + {V_C}^2} = \sqrt{IR^2 + {IX_C}^2} = I\sqrt{R^2 + {X_C}^2}$

$I = \frac{V}{\sqrt{R^2+{X_C}^2}}$ or $I = \frac{V}{Z}$; where $Z = \sqrt{R^2 + {X_C}^2}$

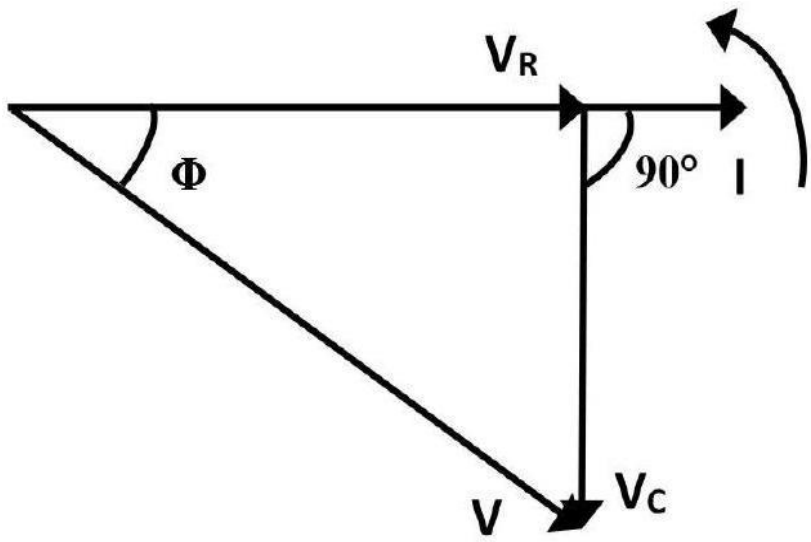
From the phasor diagram, $\tan\Phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$ or $\Phi = \tan^{-1}\frac{X_C}{R}$

From the phasor diagram, we can draw the Impedance triangle as shown in figure.

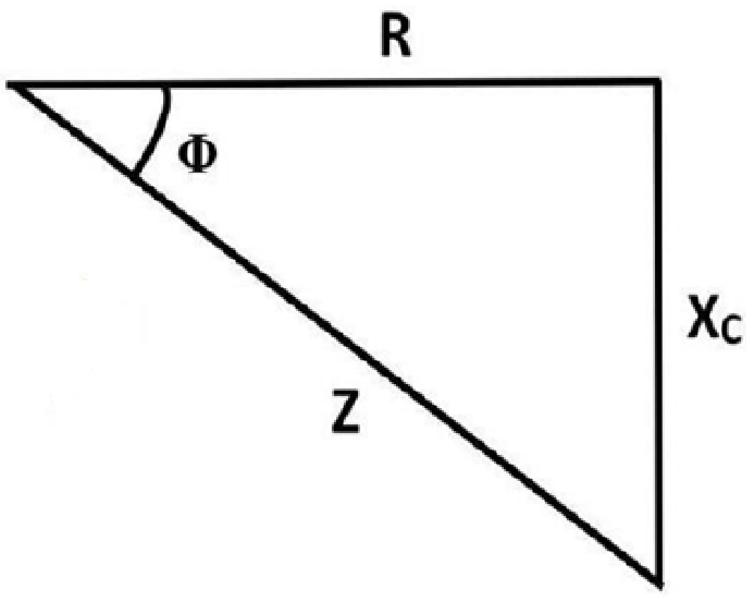
From the impedance triangle $\mathrm{Cos}\Phi = \frac{R}{Z}$
We have $P = VICos\Phi$ or $P = I^2R$



CIRCUIT DIAGRAM



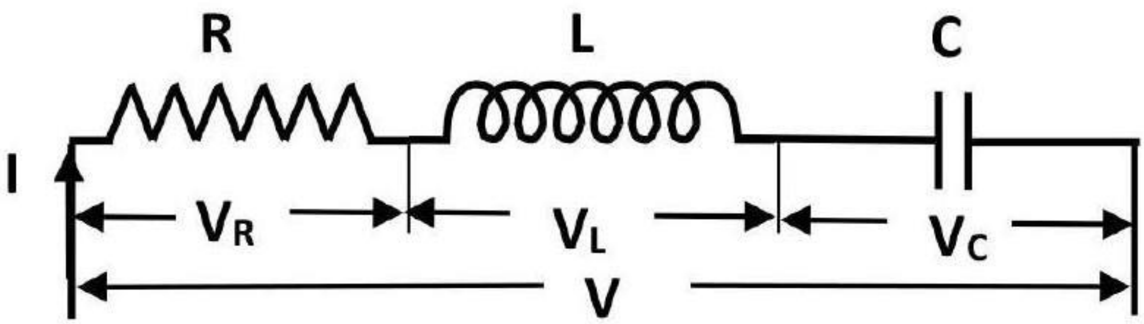
PHASOR DIAGRAM



IMPEDANCE TRIANGLE

R - L - C Series circuit:
Let us consider a resistor, inductor and capacitor in series.
If **V** is the rms value of the applied voltage then I will be the rms value of the current drawn by the circuit.

The voltage across $R = V_R = IR$, where V_R is in phase with I

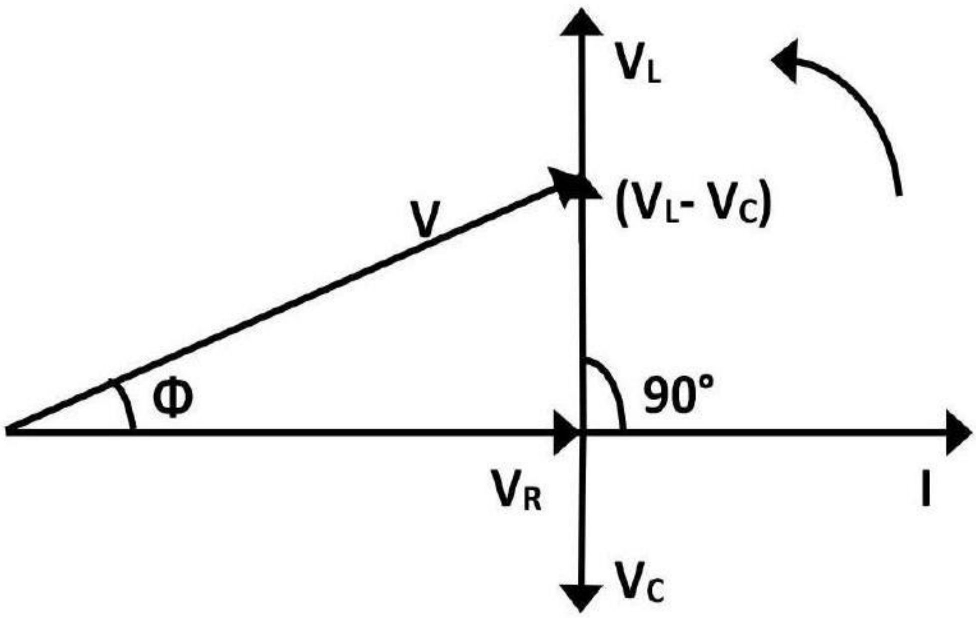


CIRCUIT DIAGRAM

The voltage across $L = V_L = IX_L$, where V_L leads I by 90°
The voltage across $\mathrm{C} = \mathrm{V_c} = \mathrm{IX}$, where $\mathrm{V_c}$ lags I by 90°
In the phasor diagram the voltages V_L and V_C are 180° out of phase with each other.
V_L is greater in magnitude than $\mathrm{V_c}$ so the resultant voltage will be ($\mathrm{V_L} - \mathrm{V_C}$).

The applied voltage V will be the phasor sum of the voltages V_R and $(V_L - V_C)$.
We have $V = \sqrt{V_R{}^2 + (V_L - V_C)^2} = \sqrt{IR^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$ or $I = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$ Or $I = \frac{V}{Z}$ where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

From the phasor diagram, $\tan\Phi = \frac{(V_L-V_C)}{V_R}$

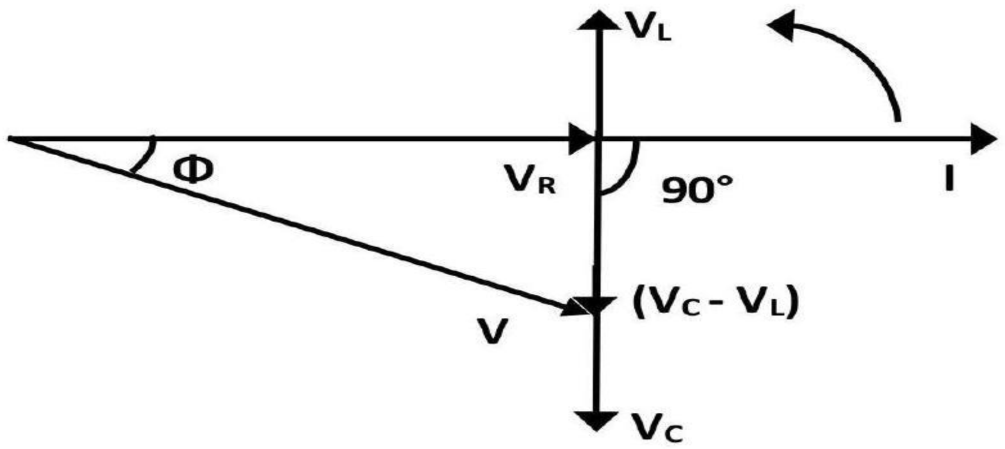


$= \frac{(IX_L - IX_C)}{IR} = \frac{(X_L - X_C)}{R}$ or $\Phi = \tan^{-1}\frac{(X_L - X_C)}{R}$

PHASOR DIAGRAM FOR $X_L > X_C$

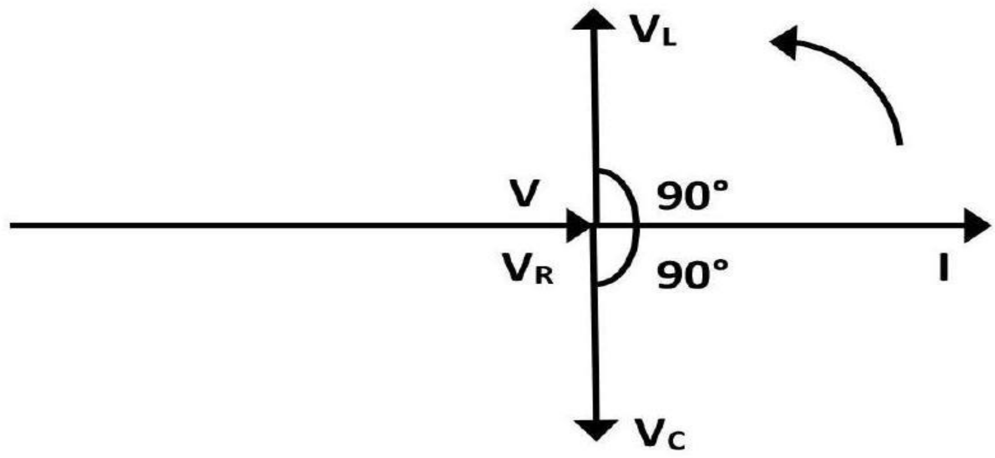
We have $\mathbf{Cos\Phi} = \frac{\boldsymbol{R}}{\boldsymbol{Z}}$ and $\boldsymbol{P} =$ VICos $\mathbf{\Phi}$ or $\boldsymbol{P} = \boldsymbol{I^2R}$

a) For the condition $X_L > X_c$ from the phasor diagram we find that the current lags the applied voltage by an angle Φ, which is greater than zero but less than $90°$. So a series RLC circuit with $X_L > X_C$ behaves as a R-L series circuit.



b) For the condition $X_C > X_L$ from the phasor diagram we find that the current leads the applied voltage by an angle Φ which is greater than zero but less than $90°$. So a series RLC circuit with $X_C > X_L$ behaves as a R-C series circuit.

c) For the condition $X_L = X_C$ from the phasor diagram we find that the current is in phase with the applied



PHASOR DIAGRAM FOR $X_L = X_C$

voltage, the phase angle between the current and applied voltage is zero. So the series RLC circuit with $X_L = X_C$ behaves as a pure R circuit. Hence a series R-L-C circuit can behave in three different ways depending upon the values of the Inductive and Capacitive reactances.

Three phase AC Circuits - Syllabus

Necessity and advantages of three phase systems, Meaning of Phase sequence, balanced and unbalanced supply and loads. Relationship between line and phase values for balanced star and delta connections. Power in balanced and unbalanced three-phase systems and their measurement.Expression for power factor

THREE PHASE CIRCUITS

Necessity of three phase systems:

Three phase power is usually generated, transmitted and distributed as it has a large number of advantages. Three phase systems are widely used by electrical grids all over the world to transfer power. They are also used to power large motors and other heavy loads. Three phase systems are always adopted because they are very economical.

Advantages of three phase systems:

The advantages of three phase systems over single phase systems are:-
Three phase apparatus are smaller in size and lighter in weight than a single phase apparatus with same power output, which makes them cheaper.
% Three phase systems require only 75% of the weight of conducting material of that required by single phase systems to transmit the same amount of power.

- Parallel operation of three phase generators is simple when compared to that of single phase generators

Output of a three phase machine is 1.5 times the output of a single phase machine of the same size.
*Three phase motors are self-starting but single phase motors are not self starting
*Three phase motors have better power factor and efficiency compared to single phase motors.

*In a single phase circuit, the power delivered is pulsating, whereas, in a three phase system, constant power is delivered when the loads are balanced.
: In a single phase system, the instantaneous power is not constant and is sinusoidal, which results in vibrations, but in a three phase system, the instantaneous power is always the same.

Single phase supply can be managed from a three phase supply, but it is not possible to get a three phase supply from a single phase supply.
*Three phase supply can be rectified into dc supply with lesser ripple factor.
Phase sequence: It is the order in which the voltages in the three coils reach their positive maximum values one after the other.

Balanced systems:

In a three phase circuit if all the impedances are equal then the circuit is called a balanced system. To a balanced circuit if balanced three phase voltage is applied the currents flowing shall also be balanced. The three phase voltages are equal in magnitude but 120° out of phase from each other and so are the three currents. The sum of all the phase currents or the phase voltages will be equal to zero.

Unbalanced systems:

The three phase systems can be unbalanced. The unbalance may be due to the unbalance in the supply or unbalance in the load. Sometimes both the supply and the load may be unbalanced.

In the analysis of unbalanced systems each phase has to be treated separately and the resultant can be obtained by their phasor representation. Let us consider a balanced supply system where the three phase voltages are: $V\angle 0^\circ, V\angle 120^\circ, V\angle 240^\circ$. Let the unbalanced impedances be : $Z_1\angle\theta_1^\circ, Z_2\angle\theta_2^\circ, Z_3\angle\theta_3{}^\circ$ respectively in the three phases.

The current in each phase will be:

$$I_1 = \frac{V}{Z_1}\angle -\theta_1{}^\circ;\ I_2 = \frac{V}{Z_2}\angle 120-\theta_2{}^\circ;\ I_3 = \frac{V}{Z_3}\angle 240-\theta_3{}^\circ$$

Star connection: If similar ends of the three phase windings are joined together at a common point N , a star connection is obtained.

Delta connection: If dissimilar ends of the three phase windings are joined to form a closed loop, a delta connection is obtained.

Relations in star connection:

From the figure - 1, it is observed that, the current flowing in the phase winding of each phase = The current flowing in that respective line. Hence, Phase current = Line current or $\boldsymbol{I}_{\mathrm{PH}} = \boldsymbol{I}_L$

Consider the lines R and Y, line voltage V_{RY} is the phasor difference of E_{RN} and E_{YN}. To subtract E_{YN} from E_{RN}, the phasor E_{YN} is reversed and the phasor sum with E_{RN} is obtained. The two phasors E_{RN} and $-\mathrm{E}_{YN}$ are equal in magnitude and equal to E_{PH} and are $60°$ apart as observed in the phasor diagram shown in figure - 2.

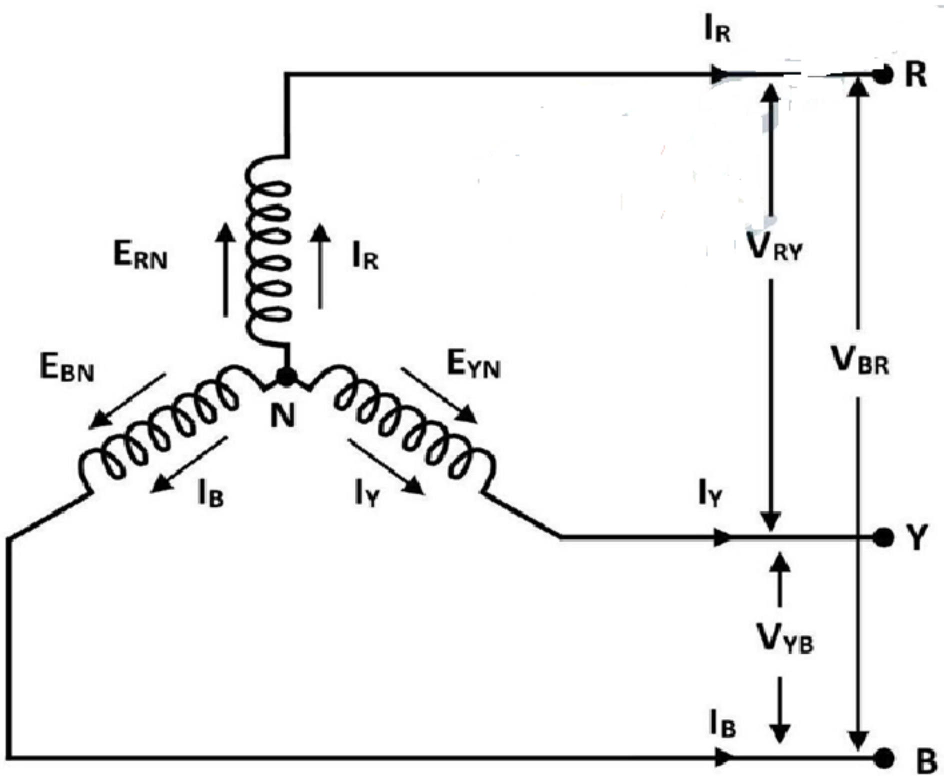


Fig. - 1

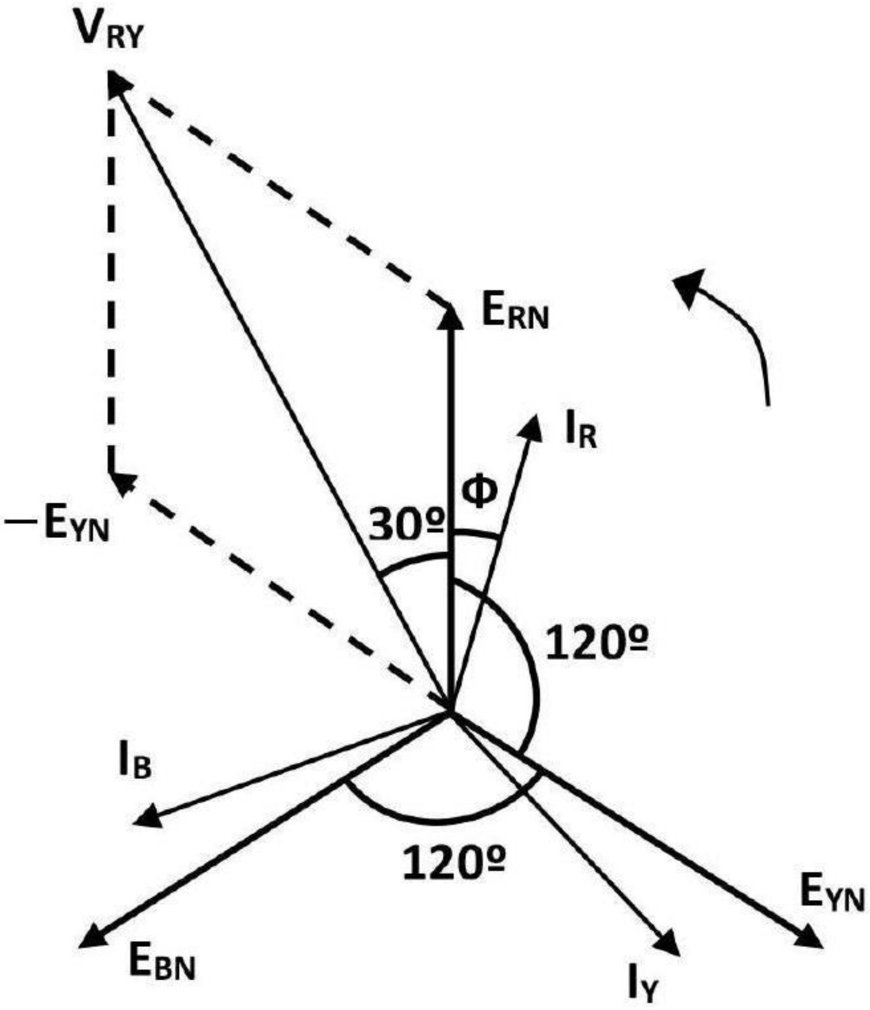


Fig. - 2

From the phasor diagram shown in figure - 2, we have

$$V_{RY} = 2E_{PH}\text{Cos}\left(\frac{60}{2}\right) = 2E_{PH}\text{Cos}30^{\circ} = \sqrt{3}E_{PH}$$

Similarly $V_{YB} = E_{YN} - E_{BN} = \sqrt{3}E_{PH}$ and $V_{BR} = E_{BN} - E_{RN} = \sqrt{3}E_{PH}$
$\therefore V_L = \sqrt{\mathbf{3}}V_{PH}$ and $_L = I_{PH}$
From figure - 2, it is observed that: The line voltages are 120° apart

The line voltages are 30° ahead of their respective phase voltages
The angle between the line currents and corresponding line voltages is ($\mathbf{30 + \Phi}$)
Power per phase $= \boldsymbol{V_{PH}I_{PH}}\mathbf{Cos\Phi}$; Total power $= \mathbf{3}\boldsymbol{V_{PH}I_{PH}}\mathbf{Cos\Phi}$
With line values, Total power $= \sqrt{3}V_L I_L \text{Cos}\Phi$ where Φ is the phase angle between V_{PH} and I_{PH}

Relations in delta connection:

From the figure - 3, it is observed that one phase winding is included between any pair of lines.

Hence, the Line voltage = Phase voltage ie. $V_L = V_{PH}$

The current in any line is equal to the phasor difference of the currents in the two phases attached to that line. Hence, the current in line R is the phasor difference of I_R and I_B. To subtract I_B from I_R, the phasor I_B is reversed and its phasor sum with $\mathrm{I_R}$ is obtained. The two phasors $\mathrm{I_R}$ and $-\mathrm{I_B}$ are equal in magnitude and equal to I_{PH} and are $60°$ apart as observed in the phasor diagram shown in figure - 4.

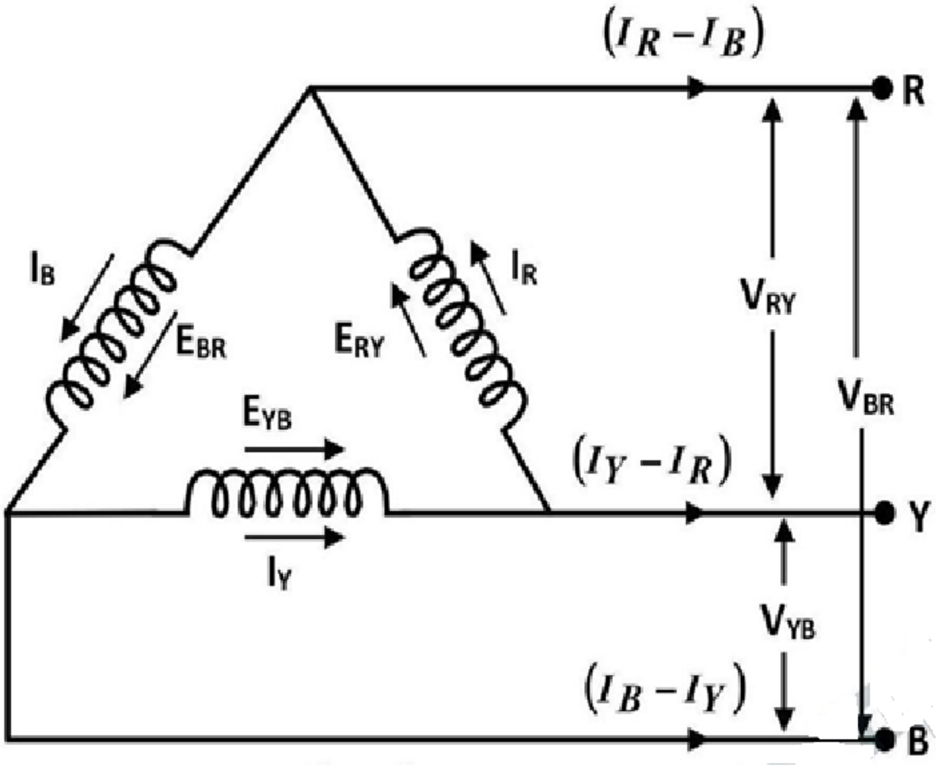


Fig. - 3

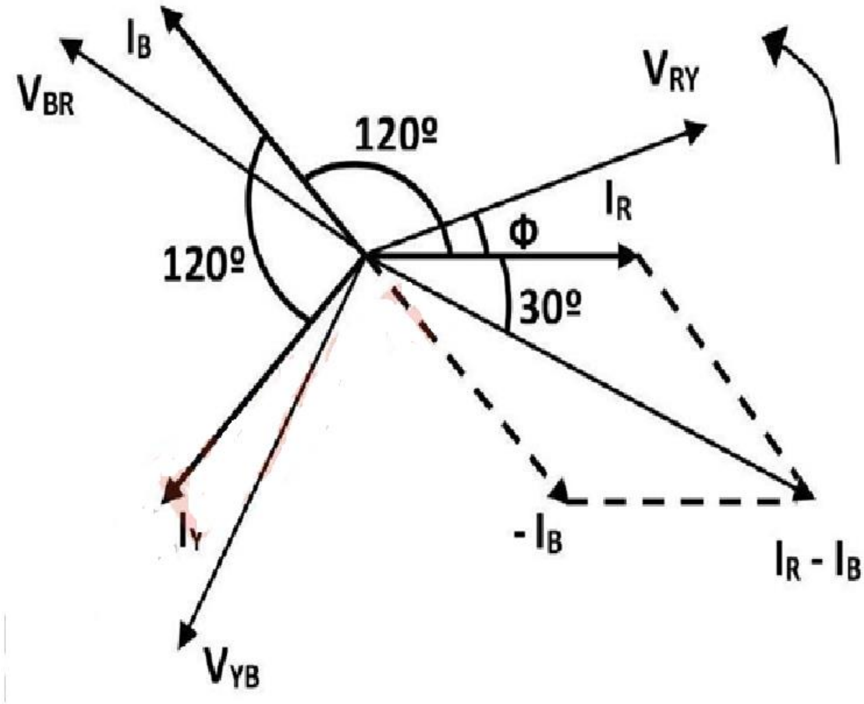


Fig. - 4

From the phasor diagram shown in figure - 4, we have

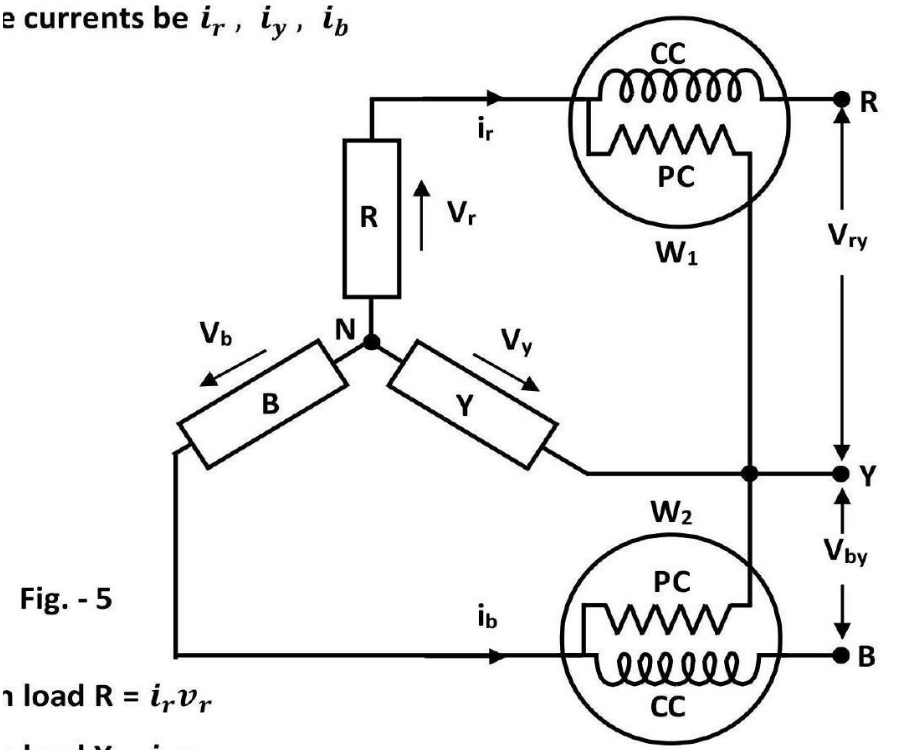
$$(I_R - I_B) = 2I_{PH}\text{Cos}\left(\frac{60}{2}\right) = 2I_{PH}\text{Cos}30^\circ = \sqrt{3}I_{PH}$$

Similarly $(I_Y - I_R) = \sqrt{3}I_{PH}$ and $(I_B - I_Y) = \sqrt{3}I_{PH}$
$\therefore I_L = \sqrt{3}I_{PH}$ and $V_L = V_{PH}$
From figure - 4, it is observed that: The line currents are 120° apart
The line currents are 30° behind their respective phase currents
The angle between the line currents and corresponding line voltages is ($\mathbf{30 + \Phi}$)
Power per phase $= V_{PH}I_{PH}\text{Cos}\Phi$; Total power $= 3V_{PH}I_{PH}\text{Cos}\Phi$
With line values, Total power $= \sqrt{3}V_L I_L \text{Cos}\Phi$ where Φ is the phase angle between V_{PH} and I_{PH}

Measurement of three phase power using two wattmeter for Star connected load:

Let us consider the loads to be connected in star as shown in figure - 5. The current coils of the two wattmeters are connected in line $\mathbf{R}$ and line $\mathbf{B}$. The potential coils of the wattmeters are connected across lines R and Y as well as lines B and Y.

Let the instantaneous values of potential difference across the loads be v_r, v_y, v_b and the corresponding values of instantaneous line currents be i_r, i_y, i_b



The instantaneous power in load $\mathrm{R} = \boldsymbol{i_r v_r}$
The instantaneous power in load $\mathrm{Y} = \boldsymbol{i_y v_y}$
The instantaneous power in load $\mathrm{B} = \boldsymbol{i_b v_b}$
Total instantaneous power $= i_r v_r + i_y v_y + i_b v_b$
From the figure - 5, it is observed that the instantaneous current through current coil of $\mathbf{W_1} = i_r$ and the instantaneous potential difference across its potential coil $=$ $(\boldsymbol{v_r - v_y})$

Instantaneous power measured by $\mathrm{W}_1 = i_r(v_r - v_y)$
Similarly the instantaneous current thro current coil of $\mathbf{W_2} = \boldsymbol{i_b}$ and the instantaneous potential difference across its potential coil $= (\boldsymbol{v_b - v_y})$

Instantaneous power measured by $W_2 = i_b(v_b - v_y)$

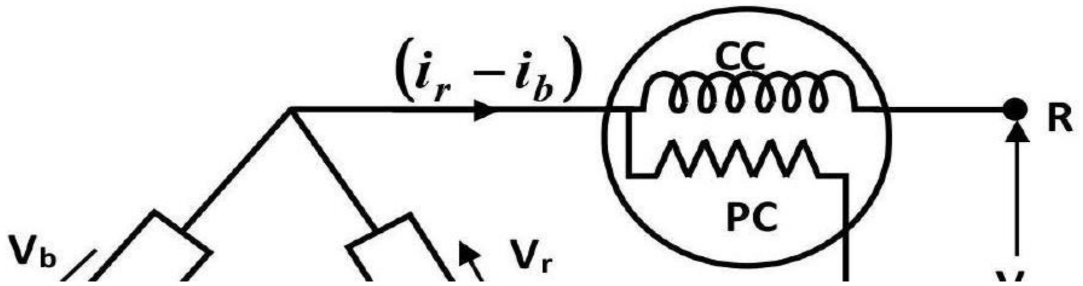
$$\mathrm{W}_1 + \mathrm{W}_2 = i_r(v_r - v_y) + i_b(v_b - v_y) \quad (1)$$

or $W_1 + W_2 = i_r v_r + i_b v_b - v_y(i_r + i_b)$

Applying KCL to the junction N shown in figure - 5 , we get
$i_r + i_y + i_b = 0$ or $(i_r + i_b) = -i_y$

Introducing equation (2) in equation (1) we get,
$W_1 + W_2 = i_r v_r + i_y v_y + i_b v_b$ which is equal to total instantaneous power consumed by a three phase star connected load.

Measurement of three phase power using two wattmeter for Delta connected load:
Let us consider the loads to be connected in delta as shown in figure - 6.
Let the instantaneous values of potential difference across the loads be v_r, v_y, v_b and the corresponding values of instantaneous phase currents be i_r, i_y, i_b



The instantaneous power in load $\mathrm{R} = \boldsymbol{i_r v_r}$
The instantaneous power in load $\mathrm{Y} = \boldsymbol{i_y v_y}$
The instantaneous power in load $\mathrm{B} = \boldsymbol{i_b v_b}$
Total instantaneous power $= i_r v_r + i_y v_y + i_b v_b$
From the figure -6 , it is observed that the instantaneous current through current coil of $W_1 = (i_r - i_b)$ and the instantaneous potential difference across its potential coil $= \boldsymbol{v_r}$

Instantaneous power measured by $\mathbf{W_1} = (\boldsymbol{i_r} - \boldsymbol{i_b})\boldsymbol{v_r}$
Similarly the instantaneous current through current coil of $\mathbf{W_2} = (\boldsymbol{i_b} - \boldsymbol{i_y})$ and the instantaneous potential difference across its potential coil $= (-v_y)$

Instantaneous power measured by $W_2 = (i_b - i_y)(-v_y)$

$$W_1 + W_2 = (i_r - i_b)v_r + (i_b - i_y)(-v_y) \qquad (1)$$

or $W_1 + W_2 = i_r v_r + i_y v_y - i_b(v_r + v_y)$

Applying KVL to closed loop ABC in figure - 6, we get

$v_r + v_y + v_b = 0$ or $(v_r + v_y) = -v_b$

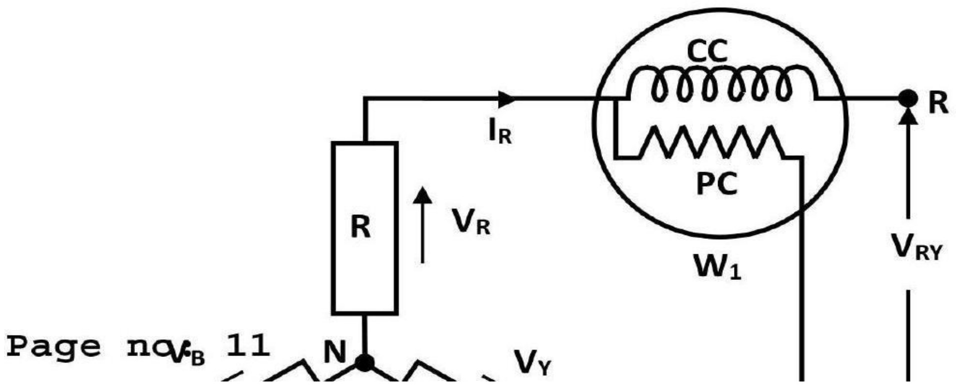
Introducing equation (2) in equation (1), we get

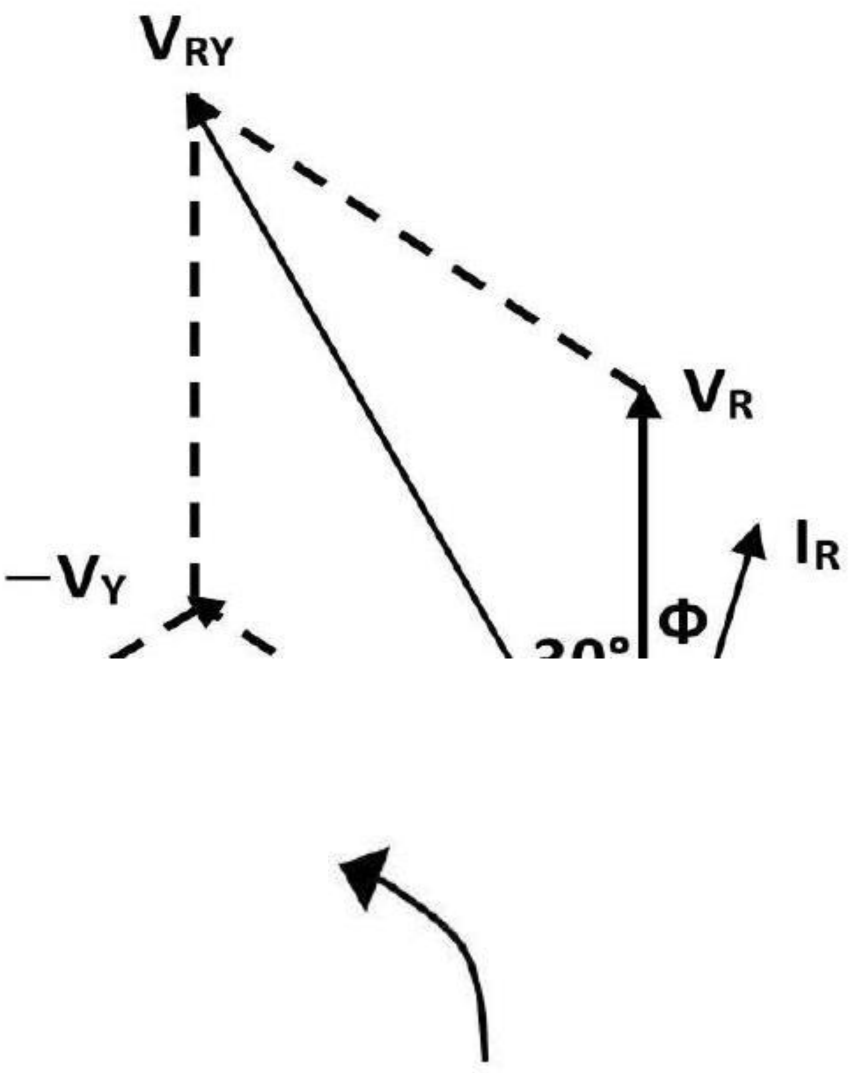
$W_1 + W_2 = i_r v_r + i_y v_y - i_b(-v_b)$

$\therefore W_1 + W_2 = i_r v_r + i_y v_y + i_b v_b$ which is equal to total instantaneous power consumed by a three phase delta connected load.

Expression for power factor in terms of wattmeter readings:

Let us consider a three phase balanced star connected load with a lagging phase angle Φ. Let V_R, V_Y and V_B be the rms values of phase voltages across the star connected load and I_R, I_Y and I_B be the phase currents.





Since the load has a lagging power factor, the phase currents lag their respective phase voltages by an angle Φ as shown in the phasor diagram. The power is measured using two wattmeters. The current through current coil of $W_1 = I_R$ and the potential difference across its potential coil $= V_{RY}$.

From the phasor diagram the phase angle between $\mathrm{V_{RY}}$ and $\mathrm{I_R} = (30 + \Phi)$
∴ Reading of $\mathbf{W_1} = \mathbf{I}_\mathrm{R}\mathbf{V}_\mathrm{RY}\mathrm{Cos}(\mathbf{30} + \boldsymbol{\Phi})$
The current through current coil of $\mathrm{W_2} = \mathrm{I_B}$ and the potential difference across its potential coil $= \mathrm{V_{BY}}$.
From the phasor diagram the phase angle between $\mathrm{V_{BY}}$ and $\mathrm{I_B} = (30 - \Phi)$.
∴ Reading of $\mathrm{W_2} = \mathrm{I_B V_{BY} Cos}(30 - \Phi)$
Since the load is balanced $I_R = I_Y = I_B = I_L$ and $V_{RY} = V_{BY} = V_{BR} = V_L$
∴ $W_1 = V_L I_L \mathrm{Cos}(30 + \Phi)$ and $W_2 = V_L I_L \mathrm{Cos}(30 - \Phi)$
$W_2 + W_1 = V_L I_L[\mathrm{Cos30Cos\Phi + Sin30Sin\Phi + Cos30Cos\Phi - Sin30Sin\Phi}]$

$$W_2 + W_1 = V_L I_L (2\text{Cos}30\text{Cos}\Phi) = V_L I_L \left(2\frac{\sqrt{3}}{2}\text{Cos}\Phi\right) = \sqrt{3} V_L I_L \text{Cos}\Phi$$

$\therefore W_2 + W_1 =$ Total Three phase power. $W_2 + W_1 = \sqrt{3}\, V_L I_L \text{Cos}\Phi$ ... (1)

$$W_2 - W_1 = V_L I_L \text{Cos}(30 - \Phi) - V_L I_L \text{Cos}(30 + \Phi) \qquad (1)$$

$$= V_L I_L [\text{Cos}30\text{Cos}\Phi + \text{Sin}30\text{Sin}\Phi - \text{Cos}30\text{Cos}\Phi + \text{Sin}30\text{Sin}\Phi]$$

$$= V_L I_L [2\text{Sin}30\text{Sin}\Phi] = V_L I_L \left[2\frac{1}{2}\text{Sin}\Phi\right] = V_L I_L \text{Sin}\Phi \qquad (2)$$

Dividing equation (2) by equation (1) we get

$\frac{(W_2 - W_1)}{(W_2 + W_1)} = \frac{(V_L I_L \sin\phi)}{(\sqrt{3} V_L I_L \cos\phi)} = \frac{\tan\phi}{\sqrt{3}}$ or $\therefore \tan\phi = \left(\sqrt{3}\frac{(W_2 - W_1)}{(W_2 + W_1)}\right)$

or $\phi = \tan^{-1}\left(\sqrt{3}\frac{(W_2 - W_1)}{(W_2 + W_1)}\right)$ $\therefore \cos\phi = \cos\left(\tan^{-1}\sqrt{3}\frac{(W_2 - W_1)}{(W_2 + W_1)}\right)$

Variation in wattmeter readings for lagging power factors:

The wattmeter readings for lagging power factors will be -

$W_1 = V_L I_L \text{Cos}(30 + \Phi)$ and $W_2 = V_L I_L \text{Cos}(30 - \Phi)$

The phase angle $\boldsymbol{\Phi}$ can be assumed to possess different values and the variation of the wattmeter readings can be observed.

Phase angle $\boldsymbol{\Phi}$	$0°$	$60°$	$90°$
Wattmeter reading W_1	+ ve	0	- ve
Wattmeter reading $\mathbf{W_2}$	+ ve	+ ve	+ ve
	$\mathbf{W_1} = \mathbf{W_2}$ for resistive loads		$W_1 = W_2$

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THANK YOU