UNIT 5

ELECTROSTATISTICS IN VACUUM

Q. 1 Explain the Coulomb's law of Electro statistics.

Ans: Coulomb proposed the laws for interaction between static electric charges. Which are as:

First Law: Like Charges repel each other and unlike charges attract each other

Second Law: Magnitude of the force exerted by the charges on each other is inversely proportional to the square of the distance between them and is directly proportional of the product of their charges

$$F \propto \frac{1}{r^2} \text{ and } F \propto q_1 q_2$$

$$F = K\frac{q_1 q_2}{r^2}$$

$$\text{Where } K = \frac{1}{4\pi\epsilon_0\epsilon_r}$$

$$\text{For air } \epsilon_r = 1$$

Q. 2 Explain the terms: Electric Field Intensity, Electric Lines of Forces and Electric Flux.

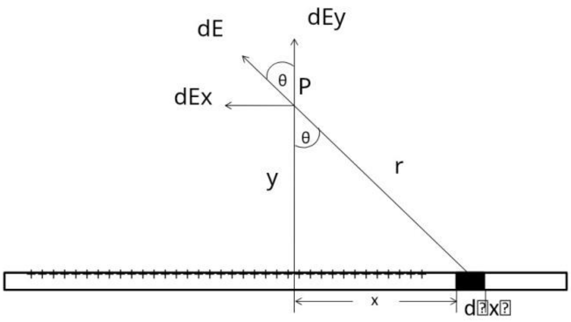
Ans: Electric Force experienced by unit positive charge in the electric filed is known as **Electric Field Intensity** and is denoted by $\mathbf{E}$. Expression for electric filed intensity is $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

Path along which a unit positive charge tend to move when free to do so in the electric filed is known **as Electric Lines of Forces.**

Number of electric lines of forces through a specified area I called as **Electric Flux** it is denoted by Greek letter ϕ. Mathematically electric flux through a surface S is given as $\phi = \int \ E \cdot ds$

Q. 3 Calculate the electric field intensity due to an infinite line charge.

Ans: Let us consider a line of infinite length having charge per unit length λ. Now we consider a point P at Y distance from line and consider a small length element dx having charge $dq = \lambda dx$, which is at a distance r from point P as shown in the figure



The x and y component of the force dE will be $dE_x = -dE\text{Sin}\theta$ and $dE_y = dE\text{Cos}\theta$

As x components at P will cancel out each other therefore net filed will be due to only y component.

Therefore

$$E = \int_{x=-\infty}^{x=+\infty} dE_y = \int_{x=-\infty}^{x=+\infty} dE\cos\theta$$

Or

$$E = \int_{x=0}^{x=+\infty} 2dE\cos\theta$$

$$E = \int_{x=0}^{x=+\infty} \frac{2dq\text{Cos}\theta}{4\pi\epsilon_0 r^2}$$

As $dq = \lambda dx$ therefore

$$E = \int_{x=0}^{x=+\infty} \frac{2\lambda dx\cos\theta}{4\pi\epsilon_0 r^2}$$

$$E = \frac{\lambda}{2\pi\epsilon_0}\int_0^{\infty} \frac{dx}{r^2}\cos\theta$$

From the figure above we can write

$$\frac{x}{y} = \tan\theta$$

Or

$$dx = y\sec^2\theta d\theta$$

Also

$$r^2 = x^2 + y^2$$

Placing the value of dx and r^2 in equation (2) we get

$$E = \frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}} \frac{y\sec^2\theta d\theta}{x^2 + y^2}\mathrm{Cos}\theta$$

Using equation (3), above equation can be rewritten as

Or

$$E = \frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}} \frac{y\sec^2\theta d\theta}{y^2\tan^2\theta + y^2}\cos\theta$$

$$E = \frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}} \frac{y\sec^2\theta\cos\theta d\theta}{y^2\sec^2\theta}$$

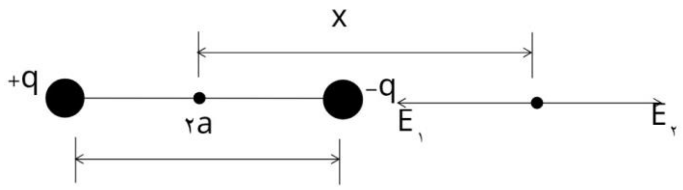
$$E = \frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{y}d\theta$$

$$E = \frac{\lambda}{2\pi\epsilon_0 y}[\sin\theta]_0^{\frac{\pi}{2}}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 y}$$

Q. 4 Calculate the electric field intensity due to a dipole at axial position.

Ans: Let us consider a dipole of separated by length $2a$. Now we consider a point P at x distance from the center of dipole as shown in the figure



Charge at P will experience two forces E_1 and E_2 in opposite directions. Where

$$E_1 = \frac{q}{4\pi\epsilon_0(x-a)^2} \text{ and } E_2 = \frac{q}{4\pi\epsilon_0(x+a)^2}$$

The resultant electric field intensity will be

$$E = E_1 - E_2$$

Or
$$E = \frac{q}{4\pi\epsilon_0(x-a)^2} - \frac{q}{4\pi\epsilon_0(x+a)^2}$$

If $x \gg a$ then

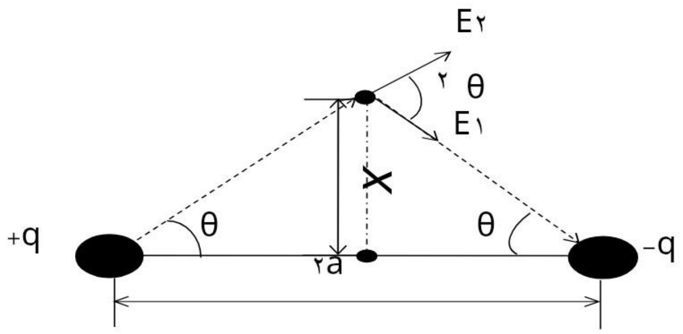
$$E = \frac{q}{4\pi\epsilon_0}\left(\frac{4xa}{x^4}\right) \Rightarrow E = \frac{4qa}{4\pi\epsilon_0 x^3} \Rightarrow E = \frac{2\times 2qa}{4\pi\epsilon_0 x^3}$$

or

$$E = \frac{2p}{4\pi\epsilon_0 x^3}$$

Q. 5 Calculate the electric field intensity due to a dipole at perpendicular bisector of dipole.

Ans: Let us consider a dipole of separated by length $2a$. Now we consider a point P at y distance from the center of dipole as shown in the figure



Electric filed at point P will be E_1 and E_2 at an angle 2θ, The magnitude of both the forces are equal i.e. we can write

$$E_1 = E_2 = \frac{q}{4\pi\epsilon_0(x^2+a^2)}$$

therefore the resultant electric filed intensity can be resolved as

$$E^2 = \left(\frac{q}{4\pi\epsilon_0(x^2+a^2)}\right)^2 + \left(\frac{q}{4\pi\epsilon_0(x^2+a^2)}\right)^2 + 2\left(\frac{q}{4\pi\epsilon_0(x^2+a^2)}\right)^2 \cos 2\theta$$

$$E^2 = 2\left(\frac{q}{4\pi\epsilon_0(x^2+a^2)}\right)^2 (1+\cos 2\theta)$$

$$E^2 = 2\left(\frac{q}{4\pi\epsilon_0(x^2+a^2)}\right)^2 (1+2\cos^2\theta - 1)$$

$$E^2 = 4\,\text{Cos}^2\theta\left(\frac{q}{4\pi\epsilon_0(x^2+a^2)}\right)^2$$

$$E = 2\cos\theta\left(\frac{q}{4\pi\epsilon_0(x^2+a^2)}\right)$$

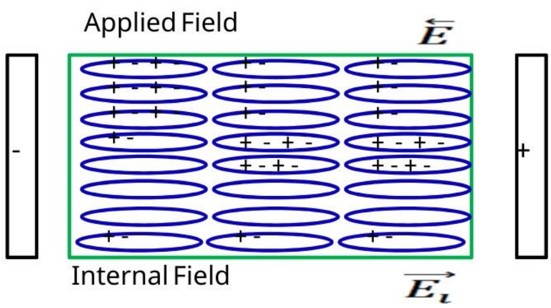
$$\text{As } \cos\theta = \left(\frac{a}{\sqrt{(x^2+a^2)}}\right)$$

$$E = \left(\frac{2aq}{4\pi\epsilon_0(x^2+a^2)^{3/2}}\right)$$

$$E = \left(\frac{P}{4\pi\epsilon_0(x^2+a^2)^{3/2}}\right)$$

Q. 6 Define the term Dielectrics, Dielectric Constant, Electric Polarization (P) and Displacement Current (D), also derive the relation between D, E and P.

Ans: Dielectrics materials are basically non conducting materials, used for storage of static charge. These materials does not allow the flow of electricity through them when placed in the electric field, but they tend to change the electric field.



Dielectric Constant or Relative Permitivity

The dielectric constant is the ratio of the permittivity of a substance to the permittivity of free space

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Dielectric constant can also be expressed as ratio of the capacitance of a capacitor with an insulating material between its plates, to its capacitance in case of vacuum between its plates.

$$\epsilon_r = \frac{C}{C_0}$$

Electric Polarization

It is defined as induced dipole moment per unit volume and is denoted by P . By definition we can write

$$\mathbf{P = ql/v}$$

Or

$$\mathbf{P = ql/Al}$$

Relation Among D, E, and P

- **E**: Electric field intensity (vector quantity)
- **P**: Polarisation vector, which is the dipole moment per unit volume
- **D**: Electric displacement vector, representing the total effect of free charges and bound charges

The relation is derived as follows:

1. The polarization vector **P** is defined as the dipole moment per unit volume due to the alignment of dipoles: **P**=unit volumedipole moment
2. The electric displacement vector **D** is defined as: **D**=ϵ0**E**+**P** where ϵ0 is the permittivity of free space.
3. This equation shows that **D** accounts for the free charges (through ϵ0**E**) and bound charges (through **P**) within the dielectric.
4. In linear, isotropic, and homogeneous dielectrics, the polarization vector **P** is proportional to **E**: **P**=ϵ0χe**E** where χe is the electric susceptibility of the material.
5. Substituting **P** back into **D**: **D**=ϵ0**E**+ϵ0χe**E**=ϵ0(1+χe)**E**=ϵ**E** where ϵ=ϵ0ϵr is the permittivity of the dielectric and ϵr=1+χe is the relative permittivity.

Q. 7 Explain the terms gradient, divergence and curl. Also give their physical significance.

Ans: Gradient: Gradient of a scalar filed is a vector quantity, whose magnitude gives the maximum space rate variation of that scalar quantity at that point. It tends to point in the direction of greatest change of scalar field. Mathematically it can be represented as

$$\text{grad} f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k = \nabla f$$

Here

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Physical significance: Gradient of potential field gives the electric field intensity i.e.

$$\vec{E} = \frac{\partial V}{\partial x}$$

Divergence: The divergence of a vector field at a point is a scalar quantity of magnitude equal to the flux of that vector field diverging out per unit volume. It tells us about the presence of source and sink within the volume under consideration. It is mathematically represented as

$$\text{div}(A) = \nabla \cdot A$$

$$\text{div}(A) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

Physical Significance: If $\text{div}(A) = 0$; then no source or sink is present in the volume under consideration it is also said as solenoid vector function. When $\text{div}(A) > 0$ it indicates the presence of source and in case $\text{div}(A) < 0$ it indicates the presence of sink.

Here

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Curl: Curl of a vector field at point is a vector quantity whose magnitude is equal to the maximum value of line integral of that vector per unit area along the boundary of a small elementary area around that point. It is given as $\text{curl}A = \nabla \times A$

$$\text{curl}A = \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix}$$

Physical Significance: Curl of a vector field tells about the rotation of the vector field. If for a vector A value of curl $A = \nabla \times A = 0$ then the vector is said to be irrotational.

Q. 8 Explain the Gauss' divergence theorem and Stokes' theorem.

Ans: Gauss's divergence theorem: This theorem states that the flux of a vector field $\vec{F}$ over any closed surface S is equal to the volume integral of the divergence of vector field over the volume enclosed by the surface S i.e.

$$\iint \vec{F} \cdot \overrightarrow{dS} = \iiint \text{div}\vec{F} \cdot \overrightarrow{dV} = \iiint (\nabla \cdot \vec{F})\overrightarrow{dV}$$

Divergence of any vector filed is defined as flux per unit volume. Mathematically we can write divergence of any vector field as

$$\text{div}\vec{F} = \frac{\iint \vec{F}\cdot\overrightarrow{dS}}{\iiint \overrightarrow{dV}}$$

Hence

$$\iint \vec{F} \cdot \overrightarrow{dS} = \iiint \text{div}\vec{F}dV$$

Stokes' theorem.

Stokes' theorem states that the net circulation of vector $\vec{F}$ over some open surface $\vec{S}$ equals to the line integral of $\vec{F}$ along the closed contour C which bounds $\vec{S}$ thus

$$\iint (\nabla \times \vec{F}) \cdot \overrightarrow{dS} = \oint \vec{F} \cdot \overrightarrow{dl}$$

In a conservative field $\oint \vec{F} \cdot \overrightarrow{dl} = 0$

By Stokes' theorem we can write

$$\iint (\nabla \times \vec{F}) \cdot \overrightarrow{dS} = 0$$

Or

$$\nabla \times \vec{F} = 0$$

Thus curl of a conservative filed is zero.

Q. 9 Give the continuity equation of current densities. Explain its physical significance.

Ans Continuity equation

Let us consider a volume V bounded by a surface S. A net charge Q exists within this region. If a net current I flows across the surface out of this region, from the principle of conservation of charge this current can be equated to the time rate of decrease of charge within this volume. Similarly, if a net current flows into the region, the charge in the volume must increase at a rate equal to the current. Thus we can write

$$I = -\frac{dQ}{dt}$$

On Applying Gauss' divergence theorem we can write,

$$\iiint \nabla \cdot J dV = -\frac{d}{dt}\iiint \rho \cdot dV$$

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$$

Equation above is known as equation of continuity.

Physical Significance

Current is the movement of charge. The **continuity equation** says that if charge is moving out of a differential volume (i.e. divergence of **current** density is positive) then the amount of charge within that volume is going to decrease, so the rate of change of charge density is negative.

Q. 10 Derive the Maxwell's equation for free space.

Ans: Maxwell's first equation

Consider a surface S bounding total charge q . Then using Gauss' law the amount of flux from the surface S can be written as

$$\iint \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

Or

$$\iint \overrightarrow{\epsilon_0 E} \cdot \vec{dS} = q$$

Or

$$\iint \vec{D} \cdot \vec{dS} = q$$

Or

$$\iint \vec{D} \cdot \vec{dS} = \iiint \rho dV$$

Using Gauss's divergence theorem

$$\iiint \nabla \cdot \vec{D} dV = \iiint \rho dV$$

On comparing both sides we can write

$$\nabla \cdot \vec{D} = \rho$$

The expression above is Maxwell's first equation.

Maxwell's second equation

As we know that isolated magnetic poles does not exist. They always exist in pairs. As a consequence magnetic lines of forces entering any arbitrary closed surface are exactly the same as leaving it. Thus flux of magnetic induction B across any closed surface is always zero.

$$\iint \vec{B} \cdot \vec{dS} = 0$$

By Gauss's divergence theorem

$$\iiint \nabla \cdot \vec{B} dV = 0$$

Or

$$\nabla \cdot \vec{B} = 0$$

Equation above is Maxwell's second equation.

Maxwell's third equation

According to Faraday's law of electromagnetic induction, the induced emf in a closed loop

$$\text{As } RGDe = -\frac{\partial \phi}{\partial t}$$

$$\phi = \iint B \cdot dS$$

Hence we can write

$$e = -\frac{\partial}{\partial t} \iint B \cdot dS$$

By definition of emf it said that emf equals the work done in carrying a unit charge around a closed loop therefore $e = \int E \cdot dl$

Or

$$\int E \cdot dl = -\frac{\partial}{\partial t} \iint B \cdot dS$$

Using Stoke's theorem

$$\iint \nabla \times E \cdot dS = -\frac{\partial}{\partial t} \iint B \cdot dS$$

Or

$$\iint \nabla \times E \cdot dS + \frac{\partial}{\partial t} \iint B \cdot dS = 0$$

$$\nabla \times E + \frac{\partial B}{\partial t} = 0$$

Or

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is Maxwell's third equation.

Maxwell's fourth equation

According to Ampere's law

$$\oint H \cdot dl = I$$

As

$$I = \iint J \cdot ds$$

Therefore

$$\oint H \cdot dl = \iint J \cdot ds$$

Using Stokes, Theorem

$$\iint \nabla \times H \cdot dS = \iint J \cdot ds$$
$$\Rightarrow \nabla \times H - \mathrm{j} = 0$$

Taking the divergence of the above equation

$$\nabla \cdot (\nabla \times H) - \nabla \cdot J = 0$$

Since divergence of curl of any vector is always zero
Therefore

$$\nabla \cdot J = 0$$

This is the case for steady fields, so equation above need to be changed for general cases.
According to Gauss' Law

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

Differentiating the equation wrt Time

$$\nabla \cdot \epsilon_0 \frac{\partial E}{\partial t} = \frac{\partial \rho}{\partial t}$$

Adding $\nabla . J$ to both sides of above equation we get

$$\nabla \cdot \mathrm{J} + \nabla \cdot \epsilon_0 \frac{\partial E}{\partial t} = \nabla \cdot \mathrm{J} + \frac{\partial \rho}{\partial t}$$

RHS of the above equation is continuity equation. Therefore

$$\nabla \cdot \mathrm{J} + \nabla \cdot \frac{\partial D}{\partial t} = 0$$

Here

$$\epsilon_0 E = D$$

So the

$$\nabla \cdot (\nabla \times H) - \nabla \cdot J = 0$$

will be rewritten as

$$\nabla \cdot (\nabla \times H) - \nabla \cdot \mathrm{J} + \nabla \cdot \frac{\partial D}{\partial t} = 0$$

Or

$$\nabla \times H = \mathrm{J} + \frac{\partial D}{\partial t}$$

Equation above is Maxwell's fourth equation.

Q. 11 Write Maxwell's equations in integral form.

Ans: Maxwell's equation in integral form.

1. **Maxwell's first equation in integral form**

$$\nabla \cdot D = \rho$$

Integrating above over an arbitrary volume

$$\iiint \nabla \cdot D dV = \iiint \rho dV$$

Using Gauss's divergence theorem

$$\iint D \cdot dS = \iiint \rho dV$$

Physical significance: It signifies that the net outward flux of electric displacement vector equals total charge within the volume

2. **Maxwell's second equation in integral form**

$$\nabla \cdot B = 0$$

Integrating above over an arbitrary volume

$$\iiint \nabla \cdot B dV = 0$$

Using Gauss's divergence theorem

$$\iint B \cdot dS = 0$$

Physical significance: It signifies that the magnetic induction flux through any closed surface is equal to zero.

3. **Maxwell's third equation in integral form**

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Integrating over a surface S bounded by a curve C

Using Stokes' theorem

$$\iint \nabla \times E \cdot dS = -\iint \frac{\partial B}{\partial t} \cdot dS$$

$$\oint E \cdot dl = -\frac{\partial}{\partial t}\iint B \cdot dS$$

The value of emf around a closed loop equals to negative rate of change of magnetic flux linked with the path.

4. **Maxwell's fourth equation in integral form**

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Integrating over a surface S bounded by a curve C

$$\iint \nabla \times H \cdot dS = \iint \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$$

Using Stokes' theorem

$$\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$$

5. The value of mmf around a closed loop equals to conduction current plus of total displacement current linked with the path.

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THANK YOU