Important Questions

UNIT-I

1. Solve : $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$
2. Solve : $(1+y^2)dx = (\tan^{-1} y - x)dy$
3. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = (1+e^x)^{-1}$
4. Solve :$\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t; x(0) = 1, y(0) = 0$
5. Solve $\cos x dy = y(\sin x - y)dx$ using Bernoulli's.
6. Solve the Linear differential equation
$$\sin 2x \frac{dy}{dx} - y = \tan x$$
7. Solve $(r + \sin\theta - \cos\theta)dr + r(\sin\theta + \cos\theta)d\theta = 0$
8. Solve the differential equation.
$$(D^3 - 7D^2 + 14D - 8)y = e^x \cos 2x$$
9. Solve $\left(D^2 - 4DD^1 + 4D^{1^2}\right)Z = \cos(x - 2y)$
10. Solve - $\left(D^2 - DD^1 - 6D^{1^2}\right)Z = xy$
11. Solve the following differential equation
$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos 2x$$
12. Solve the exact differential equation
$$ye^x dx + (2y + e^x)dy = 0$$
13. Solve $-x^2p^3 + y(1 + x^2y)p^2 + y^3p = 0$, where $p = \frac{dy}{dx}$
14. Solve $(D^3 - 3D^2 + 4)y = 0$
15. Solve the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$
16. Solve the differential equations $(D+2)(D-1)^3 y = e^x$
17. Solve $-\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{-x}$
18. Solve $-\frac{d^2y}{dx^2} + 4y = e^{-x} + \sin 2x$
19. Solve-
$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$$

20. Solve the simultaneous differential equations-

$$\frac{dx}{dt} + y = \sin t, \frac{dy}{dx} + x = \cos t$$

21. Solve $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$

22. Solve the Linear differential equation

$$(1 + y^2) + \left(x - e^{\tan^{-1}} y\right) \frac{dy}{dx} = 0$$

UNIT-II

1. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$
2. Solve $(1 - x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = x(1 - x^2)^{3/2}$
3. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ given that $x + \frac{1}{x}$ is one integral.
4. Solve the differential equation $\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} - 5y = \cos 2x$ by reducing it in normal form.
5. Solve $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + (x^2 + 1)y = x^3 + 3x$ in normal form. by changing it
6. Solve the differential equation $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$ using variation of parameter.
7. Solve $(D^2 + 4)y = \tan 2x$ by using method of variation of parameters.
8. Solve by $(D^2 + a^2)y = \tan ax$ using method of variation of parameters.
9. Solve in $(1 - x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$ series solution.
10. $x\frac{d^2y}{dx^2} - (2x - 1)\frac{dy}{dx} + (x - 1)y = 0$ if $y = e^x$ in one Integral.
11. Solve the following differential equation
$$\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = \sec x \cdot e^x$$
12. Solve $\frac{d^2y}{dx^2} - y = 0$ in series.
13. Using the method of variation of parameter solve the equation
$$\frac{d^2y}{dx^2} + y = \sec x$$
14. Using the method of variation of parameter, solve the differential equation
$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$
15. Find the general solution of $\frac{d^2y}{dx^2} + (x - 3)\frac{dy}{dx} + y = 0$ in series.
16. Show that $J_n(-x) = (-1)^n J_n(x)$ when n is a + ve or -ve integer.

UNIT-III

1. Eliminate the arbitrary function f from the relation $z = e^{xy} f(x-y)$ and form partial differential equation.
2. Form a partial differential equation by eliminating arbitrary function from :$z = f(x^2 - y^2)$
3. Solve the following differential equation - $(x^2 - y^2 - z^2)p + 2xyq = 2xz$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$
4. Solve $x^2p^2 + y^2q^2 = z^2$
5. Solve $x^2p + y^2q = (x+y)z$
6. Solve the equation $zp + yq = x$
7. Use Lagrange's method, solve the equation $xzp + yzq = xy$
8. Solve $x^2p^2 + y^2q^2 = z^2$
9. Solve $x^2p^2 + y^2q^2 = 1$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$
10. Solve the partial differential equation $$\frac{\partial^2 y}{\partial x^2} + 4\frac{\partial y}{\partial x\,\partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0$$
11. Solve $(D^2 - DD' + 2D'^2)Z = x + y$
12. Solve the partial differential equation $(D^2 - DD' - 6D'^2)z = xy$
13. Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\,\partial y} + \frac{\partial^2 z}{\partial y^2} = 12xy$
14. Solve the partial differential equation- $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\,\partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$$
15. Solve $\frac{\partial^3 z}{\partial x^3} + \frac{\partial^3 z}{\partial x^2\,\partial y} = 3x^2y$
16. Use Lagrange's method, solve the equation $yzp + zxq = xy$
17. Solve the $\left(D^3 - 3D^2D^1 + 4D^{1^3}\right)z = \mathrm{e}^{x+2y}$
18. Solve the PDE by Charpit method $z = px + qy + py$
19. Solve the PDE by Charpit method $q = 3p^2$

UNIT-IV

1. Determine whether $1/z$ is analytic or not.
2. Show that the function$e^{x}(\cos y + i\sin y)$ is analytic and find the derivative.
3. Show that the polar form of cauchy Reimann equetion

$\frac{du}{dr} = \frac{du}{rdr}$ and $\frac{du}{dr}$ = -r$\frac{du}{dr}$

And reduce that $\frac{\partial^2 u}{\partial r^2} + \frac{du}{rdr} + \frac{\partial^2 u}{r^2\,\partial\theta^2} = 0$

4. Find the analytic function u + iv of which the real part is u = $e^{x}(x\cos y - y\sin y)$.
5. Find the immaginary part of the analytice function where real part is $x^3 - 3xy^2$+ $3x^2 - 3y^2$.
6. Show that the function is $u = x^3 - 3xy^2$ harmonic and find the corresponding analytic function.
7. Using Cauchy's integral formula, find $\int_c \frac{e^{2z}}{(z+1)^3} dz$ where

   C is the curve $|z| = 2$.
8. Evaluate $\int_c \frac{1}{(z+4)z^8} dz$ where C is the circle $|z| = 2$.
9. Use Cauchy Integral formula to solve $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$
10. Using Cauchy integral theorem, to evaluate the integral $\oint_c \frac{e^{2z}}{(z-1)^2(z-3)} dz$, where C is the circle $|Z| = 2$
11. Using complex integration method,

    Solve :$\int_0^{2\pi} \frac{\cos 4\theta}{5+4\cos\theta} d\theta$
12. Show that $u(x, y) = e^{-2x}\sin 2y$ is harmonic and determine it's Harmonic conjugate.
13. By Residue theorem, $\oint_c \frac{\tan z}{z^2-1} dz$, where $C: |Z| = 2$ Evaluate
14. Evaluate the following integral using Cauchy-Integral formula

    $\oint_c \frac{4-3z}{z(z-1)(z-2)} dx$, where C is the circle $|z| = 3/2$
15. Show that the function $f(z) = e^{z}$ is analytic everywhere.
16. Evaluate $\oint_c \frac{z}{(z^2+9)} dz$, where C is the circle $|z - 2i| = 4$
17. Show that the following function is harmonic and find its harmonic conjugate functions $u = \frac{1}{2}\log(x^2 + y^2)$

UNIT-V

1. Prove that :$\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$
2. Find the directional derivative of $f(x, y, z) = e^{2x} \cos yz$ at $(0,0,0)$ in the direction of the tangent to the curve $x = a\sin t, y = a\cos t, z =$ at at $t = \frac{\pi}{4}$
3. Using Green's theorem, find the area of the region in the first quadrant bounded by the curve
$$y = x, y = \frac{1}{x}, y = \frac{x}{4}$$
4. Show that $\frac{r^{\pi}}{r^8}$ is solenoidal.
5. Verify Green's theorem for $\int_c [3x^2 - 8y^2]dx + [4y - 6xy]dy$
Where C is the region bounded by $x = 0, y = 0$ and $x + y = 1$
6. Find dir(curl $\vec{F}$) where $\vec{F} = x^2 y\hat{\imath} + nz\hat{\jmath} + 2yz\hat{n}$
7. If $\vec{\mathrm{F}} = 3xy\hat{\imath} - y^2\hat{\jmath}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.
8. Evaluate $\iint_S \vec{\mathrm{A}} \cdot \hat{n} ds$, where $\vec{\mathrm{A}} = (x + y^2)\hat{\imath} - 2x\hat{\jmath} + 2yz\hat{k}$ and S is the surface of plane $2x + y + 2z = 6$ in the first octant.
9. Find div(curl $\vec{F}$) where $\vec{F} = x^2 y\hat{\imath} + xz\hat{\jmath} + 2yz\hat{k}$
10. Using Gauss's divergence theorem, find $\iint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = (2x + 3z)\hat{\imath} - (xz + y)\hat{\jmath} + (y^2 + 2z)\hat{k}$ and S is the surface of Sphere with center $(3, -1, 2)$ and radius 3 .
11. Verify Gauss divergence theorem for $\bar{F} = x^2\bar{\imath} + y^2\bar{\jmath} + \mathrm{z}^2\mathrm{k}$ over the cube formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$.
12. Find the directional derivative of $\emptyset = xy + yz + zx$ in the direction of $2i + j + k$ at the point $(1,1,2)$. Also find the maximum value of the directional derivative at the point
13. Show that the vector $F = \frac{\vec{r}}{r^3}$ is irrotational. Find the scalar potential.
14. A vector field is given by $\vec{F} = (\sin y)i + x(1 + \cos y)j$ Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2$,
$$z = 0$$
15. Verify Stoke's theorem for the vector $\vec{F} = z\hat{\imath} + x\hat{\jmath} + y\hat{k}$ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the xy-plane.

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THANK YOU